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Contributions from SUSY-FCNC couplings to the interpretation of the HyperCP events for the decay $\Sigma^+ \to p\mu^+\mu^-$

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Abstract. The observation of three events for the decay $\Sigma^+ \to p\mu^+\mu^-$ with a dimuon invariant mass of 214.3 ± 0.5 MeV by the HyperCP Collaboration implies that a new particle X may be needed to explain the observed dimuon invariant mass distribution. We show that there are regions in the SUSY-FCNC parameter space where the A_1^0 in the NMSSM can be used to explain the HyperCP events without contradicting all the existing constraints from the measurements of the kaon decays, and the constraints from K^0–$\bar{K}^0$ mixing are automatically satisfied once the constraints from kaon decays are satisfied.

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1 Introduction

Recently, there has been a great deal of interest [1–10] in the interpretation of the observed three events for the decay $\Sigma^+ \to p\mu^+\mu^-$ with a dimuon invariant mass of 214.3 ± 0.5 MeV, which were reported by the HyperCP Collaboration [11]. The branching ratio based on the three events for this process is $(8.6^{+6.6}_{-5.4} \pm 5.5) \times 10^{-8}$ [11]. It has been argued that in the framework of the standard model (SM) it is possible to account for the total branching ratio when the long-distance contributions are properly included, but all three events that are around 214 MeV cannot be explained [11, 12]. If no new evidence supports the SM explanations in the future experiments with more events, the interpretation of the three events with the existence of a new particle, X, beyond the SM is most likely correct. However, an explanation in terms of a new particle for the HyperCP events seems too radical because there are not earlier experiments that observe such a 214 MeV new particle. If this new light particle does indeed exist and does contribute to hyperon decay, it may also contribute to the kaon and B-meson decays. So, the fact that lots of experiments at this low-energy region did not observe the new 214 MeV particle means that strong constraints have been imposed on the new particle explanation for the HyperCP events.

The authors of [8] proposed an argument to explain the HyperCP events for the hyperon decay with the new particle X without contradicting the constraints on the X from the low-energy experiments. As shown in [8], in addition to the flavor-changing two-quark contributions, there are also four-quark contributions arising from the combined effects of the usual SM $|\Delta S| = 1$ operators and the flavor-conserving couplings of X, which are comparable with the two-quark ones and cancel sufficiently to lead to suppressed rare kaon decays rates, while combining the above two kinds of contributions yields $\Sigma \to p\mu^+\mu^-$ rates within the required bounds.

Based on the analysis in [8], the authors of [7] pointed out that a light pseudoscalar Higgs particle A_1^0 in the next-to-minimal supersymmetric standard model (NMSSM) [13–15] can be identified with X. In fact, the mass of the light pseudoscalar Higgs particle A_1^0 in the NMSSM may be as small as 214 MeV in the large $\tan\beta$ limit. Under some assumptions, it has been shown in [7] that there are regions in the parameter space where A_1^0 may satisfy the following constraints:

$$\begin{aligned} \mathcal{B}\left(K^\pm \to \pi^\pm A_1^0\right) &\lesssim 8.7 \times 10^{-9}\,, \\ \mathcal{B}\left(K_s \to \pi^0 A_1^0\right) &\lesssim 1.8 \times 10^{-9}\,, \\ \mathcal{B}\left(B \to X_s A_1^0\right) &\lesssim 8.0 \times 10^{-7}\,, \end{aligned} \qquad (1)$$

which are obtained in [8] from the measurements of the kaon and B-meson decays [16–20], and these simultaneously explain the HyperCP events.

However, the author of [7] only considered contributions from the SUSY charged current, i.e., contributions arising from the exchanges of a chargino and a squark, not including contributions arising from SUSY-flavor-changing neutral currents (FCNC). It is well known that the SUSY-FCNC couplings can yield important (and, sometimes, even dominate) contributions to low-energy flavor physics, so further investigation of the possibility of the SUSY-FCNC mediating HyperCP events is needed. In

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this paper, we show that the SUSY-FCNC effects also can explain the HyperCP events and satisfy all the constraints in (1). We adopt the mass insertion method [21–24] to parameterize the flavor-changing effects and calculate SUSY-FCNC contributions to the branching ratio of $\Sigma \to pA_1^0$ and rare kaon decays. This method introduces the super-CKM basis for the quark and squark states. The couplings of quarks and squarks to the neutral gauginos are flavor diagonal, while the flavor-changing SUSY effects are exhibited in the off-diagonal terms of the squark mass matrix denoted by $(\Delta_{ij}^q)_{IJ}$, where $I, J = L, R$ and $i, j = 1, 2, 3$ indicate chiral and flavor indices respectively, and $q = u, d$ denotes the type of quark. The squark propagator is then expanded as a series of $(\delta_{ij}^q)_{IJ} = (\Delta_{ij}^q)_{IJ}/\tilde{m}^2$, where $\tilde{m}$ is the average squark mass. Using the mass insertion method, we can perform calculations of the SUSY-FCNC contributions to $\Sigma \to pA_1^0$ and rare kaon decays. Since the relevant $(\delta_{ij}^q)_{IJ}$ does not involve B-mesons decay, we do not consider the constraints from B-mesons decay. It is well known [23–29] that the parameters $(\delta_{12}^d)_{IJ}$ used in our calculations also yield important contributions to K^0–$\bar{K}^0$ mixing; however, our calculations will show that measurements of the $K_\mathrm{L} - K_\mathrm{S}$ mass difference and the indirect CP violation observable ϵ_K do not lead to more stringent constraints than the ones from kaon decays.

We organize our paper as follows. In Sect. 2 we give a brief summary of the NMSSM. In Sect. 3 we calculate the two-quark flavor-changing contributions to the Σ and kaon decays arising from the SUSY-FCNC effects mediated by neutralinos and gluinos. In Sect. 4 we combine our two-quark contributions with the four-quark contributions in [8] to give numerical results and a discussion. Feynman rules and analytical expressions for the four-quark contributions are collected in Appendices A and B, respectively.

2 NMSSM

In order to make our paper self-contained, we start with a brief description of the NMSSM and the relevant couplings considered in our paper. The superpotential of the NMSSM is given by [13–15]

$$W = QY_uH_uU + QY_dH_dD + LY_eH_dE + \lambda H_dH_uN - \frac{1}{3}kN^3\,, \tag{2}$$

where H_u and H_d form the SU(2) doublet with hypercharge $1/2$ and $-1/2$, being responsible for the up- and down-type quark mass, respectively. The ratio of the vacuum expectation values (VEVs) of H_u and H_d is defined as $\tan\beta$, just like those in the minimal supersymmetric standard model (MSSM). Compared with the MSSM, there is one more gauge-singlet Higgs field N with hypercharge 0 and VEV x in the NMSSM. After breaking of the supersymmetry, there are seven physical Higgs bosons in the NMSSM, including two charged Higgs bosons, three neutral scalar and two pseudoscalar Higgs bosons.

The Higgs potential of NMSSM is [30]

$$V_\mathrm{Higgs} = V_\mathrm{soft} + V_F + V_D, \tag{3}$$

where

$$\begin{aligned} V_\mathrm{soft} &= m_{H_d}^2|H_d|^2 + m_{H_u}^2|H_u|^2 \\ &\quad - \left(\lambda A_\lambda H_dH_uN - \frac{1}{3}kA_kN^2 + \mathrm{h.c.}\right), \\ V_F &= |\lambda|^2\left(|H_d|^2 + |H_u|^2\right)|N|^2 + \left|\lambda H_dH_u - kN^2\right|^2, \\ V_D &= \frac{g^2 + g'^2}{8}\left(|H_d|^2 - |H_u|^2\right) + \frac{g^2}{2}\left|H_u^\dagger H_d\right|^2. \end{aligned} \tag{4}$$

The above Higgs potential has a global $\mathrm{U}(1)_R$ symmetry in the limit of vanishing parameters $A_k,\ A_\lambda \to 0$ [31]. If the global $\mathrm{U}(1)_R$ symmetry is broken slightly, the lighter pseudoscalar A_1^0 has a natural small mass:

$$m_{A_1^0}^2 = 3kxA_\lambda + \mathcal{O}\left(\frac{1}{\tan\beta}\right). \tag{5}$$

In the large $\tan\beta$ limit, $m_{A_1^0}$ can be as low as $\sim 100\,\mathrm{MeV}$ [30, 31].

The Lagrangian describing the couplings of A_1^0 to the up- and down-type quarks and to the leptons are given by [7]

$$\mathcal{L}_{\mathcal{A}qq} = -(l_um_u\bar{u}\gamma_5u + l_dm_d\bar{d}\gamma_5d)\frac{\mathrm{i}A_1^0}{v} \tag{6}$$

and

$$\mathcal{L}_{\mathcal{A}\ell} = \frac{\mathrm{i}g_\ell m_\ell}{v}\bar{\ell}\gamma_5\ell A_1^0, \tag{7}$$

respectively, where

$$l_d = -g_\ell = \frac{v}{\sqrt{2}x}\left(\frac{A_\lambda - 2kx}{A_\lambda + kx}\right), \quad l_u = \frac{l_d}{\tan^2\beta}. \tag{8}$$

Note that l_u can be neglected in the large $\tan\beta$ limit. The four-quark contributions can be deduced from the interactions in (6) combined with the operators due to W exchange between the quarks [7].

It has been shown in [7] that an A_1^0 of mass 214.3 MeV decays dominantly to a muon–antimuon pair, and $\mathcal{B}(A_1^0 \to \mu^+\mu^-) \sim 1$ can be assumed. In addition, the constraint imposed by the anomalous magnetic moment of the muon is given by [4]

$$|g_\ell| \lesssim 1.2\,. \tag{9}$$

Moreover, the neutralino sector of the NMSSM is different from that in the MSSM. There are five neutralinos in the NMSSM, and the Lagrangian for the mass term of the neutralinos can be written as [15]

$$\mathcal{L}_{m_{\chi^0}} = -\frac{1}{2}(\psi^0)^\mathrm{T}Y\psi^0 + \mathrm{h.c.}, \tag{10}$$

where Y is the symmetric neutralino mixing matrix; its expression can be found in [15]. The masses of the physical neutralinos can be obtained by diagonalizing Y by a unitary 5×5 matrix N:

$$m_{\chi_i^0}\delta_{ij} = N^*_{im} Y_{mn} N_{jn}\,. \tag{11}$$

3 The SUSY-FCNC effects

The full hadronic amplitudes for the kaon decays in (1) and $\Sigma \to p A_1^0$ all can be written in the following form:

$$\mathcal{M}_{\text{full}} = \mathcal{M}_{2q} + \mathcal{M}_{4q}\,, \tag{12}$$

where $\mathcal{M}_{2q}$ arises from the SUSY-FCNC interactions at the quark level, and $\mathcal{M}_{4q}$ arises from the four-quark interactions. Using the method shown in [8], we calculate the four-quark contributions, and their expressions and numerical results are given in Appendix B. In this section, we mainly concentrate on the hadronic amplitudes $\mathcal{M}_{2q}$ from the two-quark contribution and give our analytical results. In general, there are two kinds of FCNC contributions arising from neutralino and gluino exchange, respectively. The relevant Feynman diagrams for the $s \to d$ transitions are shown in Fig. 1. Note that there are five kinds of neutralinos in the loops in the NMSSM. Calculating these Feynman diagrams, one can obtain the two-quark FCNC Lagrangian for $s \to d A_1^0$:

$$\mathcal{L}_{\mathcal{A}sd} = \mathrm{i} C_L \bar{d}\frac{1-\gamma_5}{2} s A_1^0 + \mathrm{i} C_R \bar{d}\frac{1+\gamma_5}{2} s A_1^0 + \text{h.c.}\,, \tag{13}$$

with

$$C_{L(R)} = C^{\tilde{\chi}^0}_{L(R)} + C^{\tilde{g}}_{L(R)}\,, \tag{14}$$

where $C^{\tilde{\chi}^0}_{L(R)}$ and $C^{\tilde{g}}_{L(R)}$ denote contributions from neutralino and gluino exchange, respectively. They are given by

$$\begin{aligned}
C_L^{\tilde{\chi}^0} = \frac{\alpha}{4\pi}\sum_{i,j=1}^{5} \Big\{ & (\delta^d_{12})_{LL}\big[I^1_{ij} m_d L_{2i} R^*_{2j} - I^3_i m_d L_{2i} R^*_{2i} \\
& + I^4_i m_d (m_s^2 L_{2i} R^*_{1i} - L_{1i} R^*_{2i})\big] \\
& + (\delta^d_{12})_{LR}\big[I^1_{ij} m_d m_s L_{2i} R^*_{1j} - I^3_i m_d m_s L_{2i} R^*_{1i} \\
& - I^4_i m_d m_s (L_{1i} R^*_{1i} + L_{2i} R^*_{2i})\big] \\
& + (\delta^d_{12})_{RL}\big[I^1_{ij} L_{1i} R^*_{2j} - I^3_i L_{1i} R^*_{2i} \\
& + I^4_i (m_d^2 L_{2i} R^*_{2i} + m_s^2 L_{1i} R^*_{1i})\big] \\
& + (\delta^d_{12})_{RR}\big[I^1_{ij} m_s L_{1i} R^*_{1j} - I^3_i m_s L_{1i} R^*_{1i} \\
& + I^4_i m_s (m_d^2 L_{2i} R^*_{1i} - L_{1i} R^*_{2i})\big]\Big\},
\end{aligned}$$

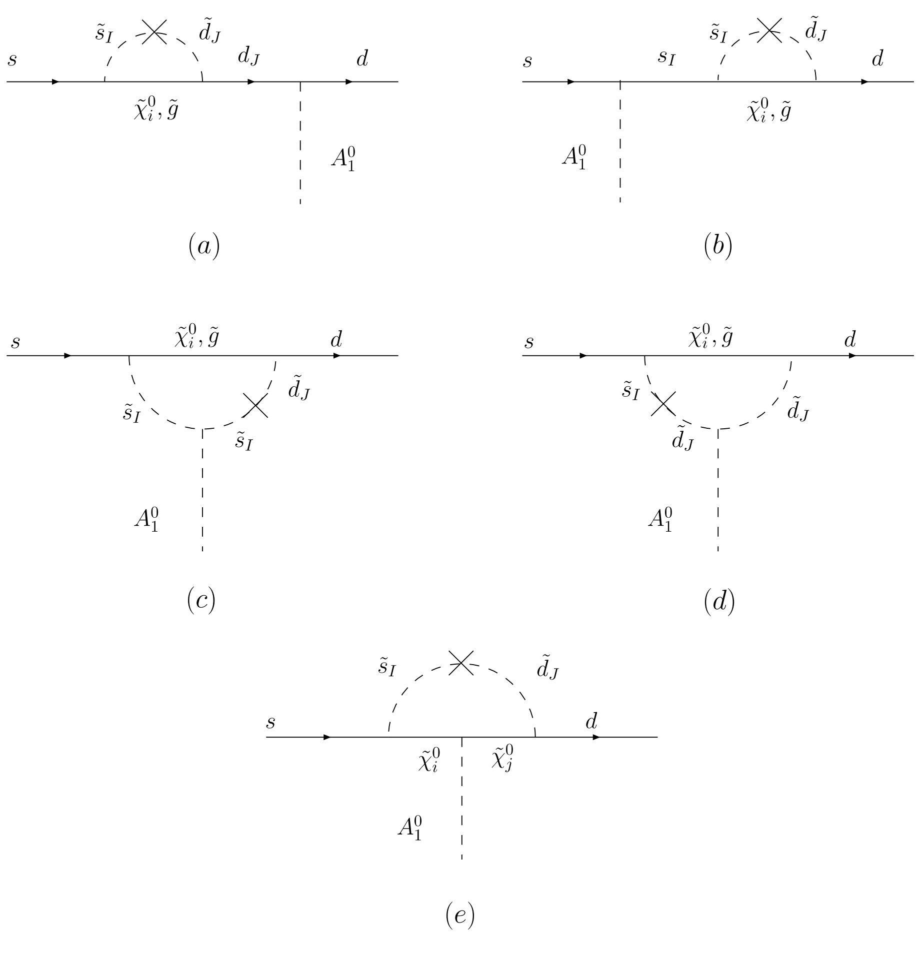

Fig. 1. Feynman diagrams for $|\Delta S| = 1$ transitions, with $I, J = L, R$, and $i, j = 1\dots 5$

$$
\begin{aligned}
C_R^{\tilde{\chi}^0} &= \frac{\alpha}{4\pi}\sum_{i,j=1}^{5}\{(\delta_{12}^d)_{LL}\left[I_{ij}^2 m_s R_{2i}L_{2j}^* + I_i^3 m_s R_{2i}L_{2i}^*\right.\\
&\quad\left. + I_i^4 m_s(R_{2i}L_{1i}^* - m_d^2 R_{1i}L_{2i}^*)\right]\\
&\quad + (\delta_{12}^d)_{LR}\left[I_{ij}^2 R_{2i}L_{1j}^* + I_i^3 R_{2i}L_{1i}^*\right.\\
&\quad\left. - I_i^4(m_d^2 R_{1i}L_{1i}^* + m_s^2 R_{2i}L_{2i}^*)\right]\\
&\quad + (\delta_{12}^d)_{RL}\left[I_{ij}^2 m_d m_s R_{1i}L_{2j}^* + I_i^3 m_d m_s R_{1i}L_{2i}^*\right.\\
&\quad\left. + I_i^4 m_d m_s(R_{2i}L_{2i}^* + R_{1i}L_{1i}^*)\right]\\
&\quad + (\delta_{12}^d)_{RR}\left[I_{ij}^2 m_d R_{1i}L_{1j}^* + I_i^3 m_d R_{1i}L_{1i}^*\right.\\
&\quad\left. + I_i^4 m_d(R_{2i}L_{1i}^* - m_s^2 R_{1i}L_{2i}^*)\right]\},\\
C_L^{\tilde{g}} &= \frac{\alpha_s}{2\pi}C_F m_{\tilde{g}}\left[2V_{Add}C_1(y_{\tilde{g}})(\delta_{12}^d)_{RL}\right.\\
&\quad\left. + V_{A\tilde{d}\tilde{d}}\frac{1}{m_{\tilde{d}}^2}D_1(y_{\tilde{g}})(m_d(\mathrm{delta}_{12}^d)_{LL} + m_s(\delta_{sd}^d)_{RR})\right],\\
C_R^{\tilde{g}} &= \frac{\alpha_s}{2\pi}C_F m_{\tilde{g}}\left[2V_{Add}C_1(y_{\tilde{g}})(\delta_{12}^d)_{LR}\right.\\
&\quad\left. - V_{A\tilde{d}\tilde{d}}\frac{1}{m_{\tilde{d}}^2}D_1(y_{\tilde{g}})(m_d(\delta_{12}^d)_{RR} + m_s(\delta_{sd}^d)_{LL})\right],
\end{aligned}
\tag{15}
$$

with

$$
\begin{aligned}
V_{Add} &= \frac{g}{2m_W\cos\beta}U_{11}^P,\\
V_{A\tilde{d}\tilde{d}} &= \frac{g}{2m_W\cos\beta}\left[\lambda(v_2U_{13}^P + xU_{12}^P) - A_D U_{11}^P\right],
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
L_{1i} &= \frac{2N_{i,1}^*}{3\sqrt{2}c_{\mathrm{W}}},\\
L_{2i} &= \frac{N_{i,3}^*}{\sqrt{2}m_W s_{\mathrm{W}}\cos\beta},\\
R_{1i} &= L_{2i}^*,\\
R_{2i} &= \frac{s_{\mathrm{W}}N_{i,1} - 3c_{\mathrm{W}}N_{i,2}}{3\sqrt{2}c_{\mathrm{W}}s_{\mathrm{W}}},
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
I_{ij}^1 &= D_2(y_i,y_j)R_{1ij}^{R''} + \frac{m_{\tilde{\chi}_i}m_{\tilde{\chi}_j}}{m_{\tilde{d}}^2}D_2(y_i,y_j)R_{1ij}^{L''},\\
I_{ij}^2 &= D_2(y_i,y_j)R_{1ij}^{L''} + \frac{m_{\tilde{\chi}_i}m_{\tilde{\chi}_j}}{m_{\tilde{d}}^2}D_2(y_i,y_j)R_{1ij}^{R''},\\
I_i^3 &= 2V_{Add}m_{\tilde{\chi}_i}C_1(y_i),\\
I_i^4 &= V_{A\tilde{d}\tilde{d}}\frac{m_{\tilde{\chi}_i}}{m_{\tilde{d}}^2}D_1(y_i),
\end{aligned}
\tag{18}
$$

$$
y_{\tilde{g}} = \frac{m_{\tilde{g}}^2}{m_{\tilde{d}}^2}, y_i = \frac{m_{\tilde{\chi}_i^0}^2}{m_{\tilde{d}}^2}, \quad i = 1\dots 5, \tag{19}
$$

where U^P is used to diagonalize the pseudoscalar Higgs mass matrices [15], the matrix N_{ij}, $i,j = 1\dots 5$ is the 5×5 unitary matrix defined in Sect. 2, $c_{\mathrm{W}} = \cos\theta_{\mathrm{W}}$, where θ_{W} is the Weinberg angle as usual, and

$$
\begin{aligned}
R_{1ij}^{''R} &= \frac{1}{2}\left[(U_{11}^P\cos\beta + U_{12}^P\sin\beta)\right.\\
&\quad\times\left(\frac{g}{c_{\mathrm{W}}}(N_{i2}N_{j3}^* + N_{j2}N_{i3}^*) - \sqrt{2}\lambda(N_{i5}N_{j4}^* + N_{j5}N_{i4}^*)\right)\\
&\quad + (U_{11}^P\sin\beta - U_{12}^P\cos\beta)\\
&\quad\left.\times\left(\frac{g}{c_{\mathrm{W}}}(N_{i2}N_{j4}^* + N_{j2}N_{i4}^*) + \sqrt{2}\lambda(N_{i5}N_{j3}^* + N_{j5}N_{i3}^*)\right)\right]\\
&\quad - \sqrt{2}kU_{13}^P\left(N_{i5}N_{j5}^* + N_{j5}N_{i5}^*\right),\\
R_{1ij}^{''R} &= -R_{1ij}^{''L*}.
\end{aligned}
\tag{20}
$$

The loop functions $C_1(x)$, $D_1(x)$ and $D_2(x,y)$ are defined as

$$
\begin{aligned}
C_1(x) &= \frac{x - 1 - x\log(x)}{(1-x)^2},\\
D_1(x) &= \frac{1 - x^2 + 2x\log(x)}{2(1-x)^3},\\
D_2(x,y) &= -\frac{1}{(1-x)(1-y)} - \frac{x\log(x)}{(1-x)^2(x-y)}\\
&\quad - \frac{y\log(y)}{(1-y)^2(y-x)}.
\end{aligned}
\tag{21}
$$

The quark level effective Lagrangian in (13) can be mapped onto the chiral Lagrangian at the leading order [8]:

$$
\begin{aligned}
\mathcal{L}_{\mathcal{A}} &= b_D\langle\bar{B}\{h_A,B\}\rangle + b_F\langle\bar{B}[h_A,B]\rangle + b_0\langle h_A\rangle\langle\bar{B}B\rangle\\
&\quad + \frac{1}{2}f_\pi^2 B_0\langle h_A\rangle + \text{h.c.},
\end{aligned}
\tag{22}
$$

where

$$
B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p\\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n\\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}
$$

represents the baryon fields, $f_\pi = 92.4\,\mathrm{MeV}$ is the pion decay constant, and $\langle\cdots\rangle \equiv \mathrm{Tr}(\cdots)$ in flavor-SU(3). Also, we have

$$
h_A = -\mathrm{i}(C_R\xi^\dagger h\xi^\dagger + C_L\xi h\xi)A_1^0, \tag{23}
$$

where

$$
h = T_6 + \mathrm{i}T_7 = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0 \end{pmatrix};
$$

this is used to specify the $s \to d$ transition,

$$
\xi = \mathrm{e}^{\mathrm{i}\pi/f_\pi}, \quad \Sigma = \xi\xi = \mathrm{e}^{2\mathrm{i}\pi/f_\pi}, \tag{24}
$$

and

$$
\pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}
$$

is the pion octet.

The two-quark amplitude $\mathcal{M}_{2q}$ can be deduced from $\mathcal{L}_{\mathcal{A}}$ plus the usual chiral Lagrangian $\mathcal{L}_s$ for the strong interactions of hadrons, which is expressed by [32–35]

$$\begin{aligned}\mathcal{L}_s = & \,\mathrm{i}\langle \bar{B}\gamma^\mu D_\mu B\rangle - m_0\langle \bar{B}B\rangle + D\langle \bar{B}\gamma^\mu\gamma_5\{A_\mu, B\}\rangle \\ & + F\langle \bar{B}\gamma^\mu\gamma_5[A_\mu, B]\rangle + b_D\langle \bar{B}\{M_+, B\}\rangle \\ & + b_F\langle \bar{B}[M_+, B]\rangle + b_0\langle M_+\rangle\langle \bar{B}B\rangle \\ & + \frac{1}{4}f_\pi^2\langle \partial^\mu \Sigma^\dagger \partial_\mu \Sigma\rangle + \frac{1}{2}f_\pi^2 B_0\langle M_+\rangle\,, \end{aligned} \quad (25)$$

with

$$\begin{aligned} D^\mu B &= \partial^\mu B + [V^\mu, B], \\ A^\mu &= \frac{\mathrm{i}}{2}(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi)\,, \quad M_+ = \xi^\dagger M\xi^\dagger + \xi M^\dagger\xi\,, \\ V^\mu &= \frac{1}{2}(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi)\,, \end{aligned}$$

where $M = \mathrm{diag}(\hat{m}, \hat{m}, m_s)$ is the quark mass matrix in the $m_u = m_d = \hat{m}$ limit.

Using the mass relations $m_\Sigma - m_p = 2(b_D - b_F)(m_s - \hat{m})$, $m_K^2 - m_\pi^2 = B_0(m_s - \hat{m})$ and $m_K^2 = B_0(m_s + \hat{m})$, the amplitudes for the different decay modes can be written as [8]

$$\begin{aligned}\mathcal{M}_{2q}(K^+ \to \pi^+ A_1^0) &= -\sqrt{2}\mathcal{M}_{2q}(K^0 \to \pi^0 A_1^0) \\ &= \mathrm{i}\left(\frac{C_L + C_R}{2}\right) B_0\,, \end{aligned} \quad (26)$$

$$\begin{aligned}\mathcal{M}_{2q}(\Sigma^+ \to p A_1^0) = & \,\mathrm{i}\left(\frac{C_L + C_R}{2}\right)\frac{B_0(m_\Sigma - m_p)}{m_K^2 - m_\pi^2}\bar{p}\Sigma^+ \\ & + \mathrm{i}(D - F)\left(\frac{C_L - C_R}{2}\right) \\ & \times \frac{B_0(m_\Sigma + m_p)}{m_K^2 - m_{A_1^0}^2}\bar{p}\gamma_5\Sigma^+\,, \end{aligned} \quad (27)$$

where $B_0 = 2031\,\mathrm{MeV}$ [4].

4 Numerical results and discussion

In this section we present our numerical results. The Higgs sector of NMSSM is described by the six independent parameters λ, k, A_λ, A_k, $\tan\beta$ and μ, where $\mu = -\lambda x$. For convenience, we will take $m_{A_1^0}$ and the coupling of down-type quarks to A_1^0, l_d instead of k and λ. We follow [7] in setting $l_d = 0.35$, $-\lambda x = 150\,\mathrm{GeV}$ and $\tan\beta = 30$. A_k and A_λ are set 0.001 and 0.002, respectively. The mass of A_1^0 is set 214.3 MeV to satisfy the HyperCP data. We set the mass of gluino and average down-type squark mass 200 GeV and 350 GeV, respectively. With our input, the masses of the neutralinos are around 100–800 GeV. Numerically, contributions from the exchange of the gluinos and squarks are larger than contributions from the exchange of the neutralinos and squarks, i.e., $C_{L,R}^{\tilde{g}}$ are larger than $C_{LR}^{\tilde{\chi}^0}$ in most regions of parameter space. This is due to the effects of α_s.

Our numerical results are shown in Figs. 2–6. We first assume that the $(\delta^{12})_{IJ}$ are real. The allowed regions in parameter space are shown in Figs. 2–4, where the gray areas are the allowed regions for A_1^0 to explain the HyperCP events. When the constraints obtained from the kaon decays are considered, the allowed parameter space

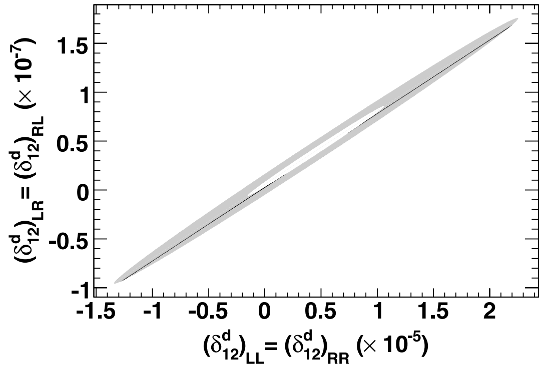

Fig. 2. The allowed values of $(\delta_{12}^d)_{LR} = (\delta_{12}^d)_{RL}$ as a function of $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR}$. The *gray area* is the region where A_1^0 can explain the HyperCP events, when the constraints from rare kaon decays are considered; the allowed region is greatly reduced to the *dark one*

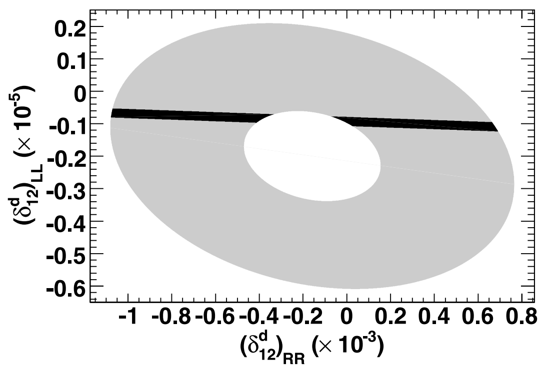

Fig. 3. The allowed values of $(\delta_{12}^d)_{LL}$ as a function of $(\delta_{12}^d)_{RR}$, The *gray area* is the region where A_1^0 can explain the HyperCP events, when the constraints from rare kaon decays are considered; the allowed region is greatly reduced to the *dark one*

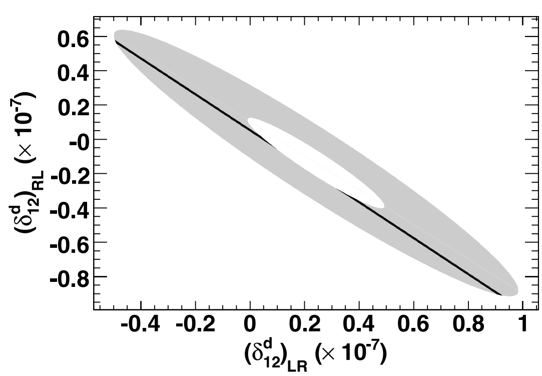

Fig. 4. The allowed values of $(\delta_{12}^d)_{RL}$ as a function of $(\delta_{12}^d)_{LR}$, The *gray area* is the region where A_1^0 can explain the HyperCP events, when the constraints from rare kaon decays are considered; the allowed region is greatly reduced to the *dark one*

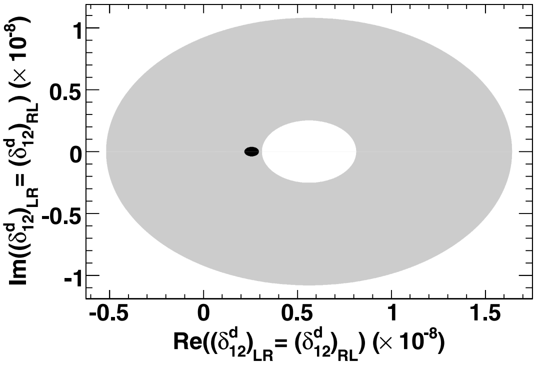

Fig. 5. The allowed values of $\mathrm{Im}\{(\delta^d_{12})_{LR} = (\delta^d_{12})_{RL}\}$ as a function of $\mathrm{Re}\{(\delta^d_{12})_{LR} = (\delta^d_{12})_{RL}\}$. The *gray region* denotes the survival region for explaining the HyperCP events alone. In the *dark region*, the A^0_1 can explain the HyperCP events and simultaneously satisfy the bounds originating from kaon decays and K^0–$\bar{K}^0$ mixing

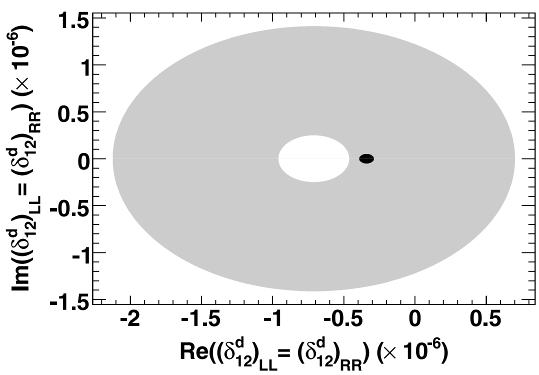

Fig. 6. The allowed values of $\mathrm{Im}\{(\delta^d_{12})_{LL} = (\delta^d_{12})_{RR}\}$ as a function of $\mathrm{Re}\{(\delta^d_{12})_{LL} = (\delta^d_{12})_{RR}\}$. The *gray region* denotes the survival region for explaining the HyperCP events alone. In the *dark region*, the A^0_1 can explain the HyperCP events and simultaneously satisfy the bounds originating from kaon decays and K^0–$\bar{K}^0$ mixing

is greatly reduced to the dark regions. From Figs. 2–4, the constraints on the combinations of the parameters can be obtained as follows:

$$\begin{aligned}
&(\delta^d_{12})_{LL(RR)}(\delta^d_{12})_{LR(RL)} \leq \\
&\quad 3.9\times10^{-12} \quad \text{(without kaon bounds)}\,, \\
&\quad 3.7\times10^{-12} \quad \text{(with kaon bounds)}\,, \\
&(\delta^d_{12})_{LL}(\delta^d_{12})_{RR} \leq \\
&\quad 3.9\times10^{-9} \quad \text{(without kaon bounds)}\,, \\
&\quad 1.8\times10^{-14} \quad \text{(with kaon bounds)}\,, \\
&(\delta^d_{12})_{LR} \quad (\delta^d_{12})_{RL} \leq \\
&\quad 2.7\text{–}16 \quad \text{(without kaon bounds)}\,, \\
&\quad 0.9\times10^{-17} \quad \text{(with kaon bounds)}\,. \qquad (28)
\end{aligned}$$

It has been widely studied in the literature that SUSY-FCNC effects have a great impact on K^0–$\bar{K}^0$ mixing if the relevant $(\delta^d_{12})_{IJ}$ are complex. So we further investigate the possible constraints on $(\delta^d_{12})_{IJ}$ from the $K_\mathrm{L} - K_\mathrm{S}$ mass difference Δm_K and the indirect CP violation parameter ϵ_K. However, the constraints shown in (28) are roughly several orders smaller than those given in the literature involving the SUSY-FCNC mediated K^0–$\bar{K}^0$ mixing [23–29], where $(\delta^d_{12})_{LL}$ and $(\delta^d_{12})_{LR(RL)}$ are around $\mathcal{O}(10^{-1}$–$10^{-3})$ and $\mathcal{O}(10^{-3}$–$10^{-4})$, respectively. This fact indicates that the constraints from K^0–$\bar{K}^0$ mixing may be automatically satisfied once the constraints from $\Sigma \to pA^0_1$ and the rare kaon decays in (1) are satisfied. Our numerical results do indeed confirm that the constraints from K^0–$\bar{K}^0$ mixing do not lead to more stringent constraints than the ones given from the kaon decays in (1) .

Figures 5 and 6 show the constraints on the complex $(\delta^d_{12})_{IJ}$ from $\Sigma \to pA^0_1$ and rare kaon decays in (1). The corresponding constraints on the combinations of parameters are given by

$$\begin{aligned}
&\mathrm{Re}(\delta^d_{12})_{LR(RL)}\mathrm{Im}(\delta^d_{12})_{LR(RL)} \leq \\
&\quad 2.7\times10^{-16} \quad \text{(without kaon bounds)}\,, \\
&\quad 1.5\times10^{-18} \quad \text{(with kaon bounds)}\,, \\
&\mathrm{Re}(\delta^d_{12})_{LL(RR)}\mathrm{Im}(\delta^d_{12})_{LL(RR)} \leq \\
&\quad 1.8\times10^{-12} \quad \text{(without kaon bounds)}\,, \\
&\quad 1.8\times10^{-14} \quad \text{(with kaon bounds)}\,. \qquad (29)
\end{aligned}$$

From Figs. 5 and 6, it can be seen that the gray areas are the allowed regions of the SUSY-FCNC parameters for A^0_1 to explain the HyperCP events and the gray regions are greatly reduced to the dark ones when the constraints from rare kaon decays are considered. Even so, there are still regions in the SUSY-FCNC parameter space, where A^0_1 in the NMSSM can be used to explain the HyperCP events without contradicting the constraints from the rare kaon decays and K^0–$\bar{K}^0$ mixing.

In conclusion, we have calculated the two-quark contributions to the decay $\Sigma^+ \to p\mu^+\mu^-$ arising from the transition $s \to dA^0_1$ via the SUSY-FCNC couplings. Combining the two-quark contributions with the four-quark contributions, we show that there are regions in the SUSY-FCNC parameter space where the A^1_0 in the NMSSM can be identified with a new particle of mass 214.3 MeV, X, which can be used to explain the HyperCP events while satisfying all the constraints from the measurements of the rare kaon decays. Once the constraints from the kaon decays are satisfied, the constraints from K^0–$\bar{K}^0$ mixing are automatically satisfied.

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Appendix A

We give the Feynman rules used in our calculations in Fig. 7, where $V_{A\tilde{d}\tilde{d}}$, V_{Add}, $L(R)_{1(2)i}$ and $R^{L(R)''}_{1ij}$ are defined

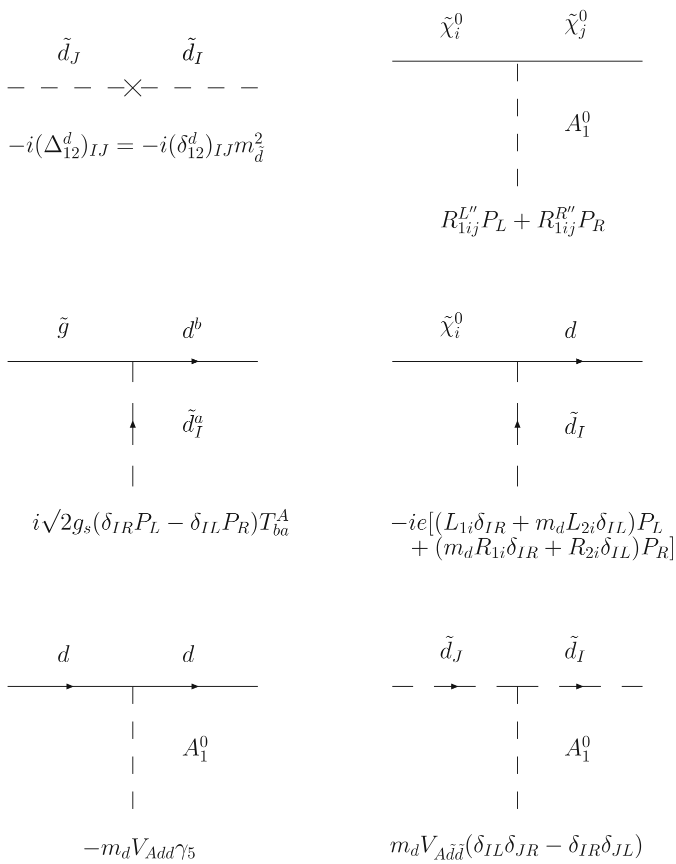

Fig. 7. Feynman rules used in our paper

in (16), (17) and (20), respectively. Also we have

$$\delta_{IJ} = \begin{cases} 1 & I = J\,, \\ 0 & I \neq J\,. \end{cases}$$

Appendix B

We collect expressions of the four-quark amplitudes for the different decay modes in this appendix. A detailed description can be found in [8]; we cite the results of this reference here. As pointed out in [7], the couplings of A^0_1 to the up-type quarks tend to zero in the limit of large $\tan\beta$, so we neglect terms that are proportional to l_u.

The four-quark contributions for the kaon decays are as follows:

$$\begin{aligned}
\mathcal{M}_{4q}(K^+ \to \pi^+ A^0_1) &= \mathrm{i}\frac{l_d\gamma_8}{v} \\
&\Bigg\{ -\frac{m^2_\pi}{2} + \left[\left(2m^2_K + m^2_\pi - 3m^2_{A^0_1}\right)c_\theta - \sqrt{8}\left(m^2_K - m^2_\pi\right)s_\theta\right] \\
&\times \frac{\left(4m^2_K - 3m^2_\pi\right)c_\theta + \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)s_\theta}{6\left(m^2_\eta - m^2_{A^0_1}\right)} \\
&+ \left[\left(2m^2_K + m^2_\pi - 3m^2_{A^0_1}\right)s_\theta + \sqrt{8}\left(m^2_K - m^2_\pi\right)c_\theta\right] \\
&\frac{\left(4m^2_K - 3m^2_\pi\right)s_\theta - \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)c_\theta}{6\left(m^2_{\eta'} - m^2_{A^0_1}\right)} \Bigg\}\,, \qquad \text{(B.1)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{4q}\left(K^0 \to \pi A^0_1\right) &= \mathrm{i}\frac{l_d\gamma_8}{v} \Bigg\{ -\frac{\left(2m^2_K - m^2_\pi - m^2_{A^0_1}\right)m^2_\pi}{\sqrt{8}\left(m^2_{A^0_1} - m^2_\pi\right)} \\
&+ \left[\left(2m^2_K + m^2_\pi - 3m^2_{A^0_1}\right)c_\theta - \sqrt{8}\left(m^2_K - m^2_\pi\right)s_\theta\right] \\
&\times \frac{\left(4m^2_K - 3m^2_\pi\right)c_\theta + \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)s_\theta}{6\sqrt{2}\left(m^2_{A^0_1} - m^2_\eta\right)} \\
&+ \left[\left(2m^2_K + m^2_\pi - 3m^2_{A^0_1}\right)s_\theta + \sqrt{8}\left(m^2_K - m^2_\pi\right)c_\theta\right] \\
&\times \frac{\left(4m^2_K - 3m^2_\pi\right)s_\theta - \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)c_\theta}{6\sqrt{2}\left(m^2_{A^0_1} - m^2_{\eta'}\right)} \Bigg\}\,, \qquad \text{(B.2)}
\end{aligned}$$

where $\gamma_8 = -7.8 \times 10^{-8}$. s_θ and c_θ are shortcuts for $\sin\theta$ and $\cos\theta$, where $\theta = -19.7°$. The four-quark contributions to the $\Sigma \to pA^0_1$ process are

$$\mathcal{M}_{4q}(\Sigma^+ \to pA^0_1) = \mathrm{i}\bar{p}(A_{pA^0_1} - B_{pA^0_1}\gamma_5)\Sigma^+\,, \qquad \text{(B.3)}$$

with

$$\begin{aligned}
A_{pA^0_1} = l_d \frac{f_\pi}{v}\frac{A_{p\pi^0}}{2} &\Bigg\{ \frac{m^2_\pi}{m^2_{A^0_1} - m^2_\pi} \\
&+ \frac{\left(4m^2_K - 3m^2_\pi\right)c^2_\theta + \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)c_\theta s_\theta}{m^2_\eta - m^2_{A^0_1}} \\
&+ \frac{\left(4m^2_K - 3m^2_\pi\right)s^2_\theta - \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)c_\theta s_\theta}{m^2_{\eta'} - m^2_{A^0_1}} \Bigg\}
\end{aligned} \qquad \text{(B.4)}$$

and

$$\begin{aligned}
B_{pA^0_1} = l_d \frac{f_\pi}{v}\frac{B_{p\pi^0}}{2} &\Bigg\{ \frac{m^2_\pi}{m^2_{A^0_1} - m^2_\pi} \\
&+ \frac{\left(4m^2_K - 3m^2_\pi\right)c^2_\theta + \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)c_\theta s_\theta}{m^2_\eta - m^2_{A^0_1}} \\
&+ \frac{\left(4m^2_K - 3m^2_\pi\right)s^2_\theta - \sqrt{2}\left(2m^2_K - \tilde{m}^2_0\right)c_\theta s_\theta}{m^2_{\eta'} - m^2_{A^0_1}} \Bigg\}\,,
\end{aligned} \qquad \text{(B.5)}$$

where $A_{p\pi^0} = -3.25 \times 10^{-7}$, $B_{p\pi^0} = 26.67 \times 10^{-7}$.

Numerically, the above amplitudes are

$$\begin{aligned}
\mathcal{M}_{4q}\left(\Sigma \to pA^0_1\right) &= \mathrm{i}\bar{p}\Bigg(-6.96 \times 10^{-7} l_d \frac{f_\pi}{v} \\
&\quad - (5.71 \times 10^{-6}) l_d \frac{f_\pi}{v}\gamma_5 \Bigg)\Sigma^+\,, \\
\mathcal{M}_{4q}(K^+ \to \pi^+ A^0_1) &= -\mathrm{i}1.08 \times 10^{-7} l_d \frac{m^2_K}{v}\,, \\
\mathcal{M}_{4q}(K^0 \to \pi A^0_1) &= \mathrm{i}1.12 \times 10^{-7} l_d \frac{m^2_K}{v}\,. \qquad \text{(B.6)}
\end{aligned}$$

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